Multifractality of agro-meteorological time series

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Sofia, October 24-28, 2016
Outline

- Premises
- Multifractal method (MF-DFA)
- Climate dynamics analysis across European transect
- Effect of spatial averaging on multifractal properties of meteorological time series
- Effect of temporal averaging on multifractal properties of meteorological time series
- Conclusions and discussion

Locations of weather stations used in this study superimposed on environmental zones as defined by Metzger et al. (2005). NRW - North Rhine-Westphalia
The aim of the FACCE MACSUR Knowledge Hub is to initiate and enhance cooperation among European researchers in linking existing models of crop production, livestock farming, farm economics and trade models to assess the impact of climate change on food security in Europe. The project, comprising 242 people working at 83 institutions or units from 70 independent organizations in 17 countries, started in June 2012.

TO ASSESS CLIMATE CHANGE IMPACT ON AGRICULTURAL SYSTEMS THE USE OF HISTORICAL LONG TERM TIME SERIES IS REQUIRED AS INPUTS TO THE MODELS.
So drastic changes of mean temperatures do not occur in Europe. But maybe long distance correlations change?

Does the climate behave like a sick organism?

Prehistoric data confirm multifractality of long distance correlations of temperature. Will the shorter time scale give similar results?
A **multifractal** is a set of intertwined fractals. Self-similarity of multifractals is scale dependent (spectrum of dimensions).

### Stages of multifractal analysis

1. Non Gaussian process
2. Variable
3. Decomposition of the time scale
4. Estimation of the distribution with stable order
5. Estimation of the multifractal spectrum (specificity spectrum) (e.g. MF-DFA, WTMM)
6. Identification of multifractal interactions through the analysis of the spectrum

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Multifractal Detrended Fluctuation Analysis

In general, the MFDFA for nonstationary time series \( x_k \) of length \( N \) consists of 5 steps (Kantelhardt et al. 2002):

1. The noises are converted into random walks by subtracting the mean value and integrating the time series.
2. Each profile is divided into \( N_s = \text{int}(N/s) \) non-overlapping segments of equal length \( s \). Since the length \( N \) of the series is often not a multiple of the considered time scale \( s \), a short part at the end of the profile may remain. The same procedure was therefore repeated starting from the opposite end to obtain two \( N_s \) segments altogether.
3. The linear local trend for each of \( 2N_s \) segments is calculated by a least square fit of the series and the variance \( F^2(s, \nu) \) is determined subsequently.
4. All segments are averaged to obtain the \( q \)th-order fluctuation function \( F_q(s) \).
5. The scaling behavior of the fluctuation functions is determined by analyzing log-log plots \( F_q(s) \) versus \( s \) for each value of \( q \). For multifractal time series, \( F_q(s) \) increases (for large values of \( s \)), as a power law \( F_q(s) \sim s^{h(q)} \) with the generalised Hurst exponent \( h(q) \) depending on \( q \). The multifractal spectrum is obtained using the relationship \( \tau(q) = qh(q) - 1 \) and using the Legendre transform \( f(\alpha) = q\alpha - \tau(q) \), where \( \alpha = d\tau/dq \).
Main parameters of a multifractal spectrum

\( \alpha_0 \):
lower value - underlying process is correlated and loses fine structure, becoming more regular in appearance

\( a_s \):
left-skewed spectrum means low fractal exponents of small weights, which correspond to the dominance of extreme events
right-skewed spectrum denotes relatively strongly weighed high fractal exponents, which correspond to fine structures

\( w \):
“richness” of the signal structure (i.e. more developed multifractality)

2 possible sources of multifractality:

a) due to a broadness of probability density function (PDF)
b) due to a different correlations in small and large scale fluctuations

As indicated by Kantelhardt et al. (2002)
Climate dynamics analysis across European transect

<table>
<thead>
<tr>
<th>Station</th>
<th>Country</th>
<th>Series length</th>
<th>Type of the data</th>
<th>Analyzed quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lublin, city center</td>
<td>Poland</td>
<td>31 years</td>
<td>Measured every hour, averaged to daily</td>
<td>R, W, AT, H</td>
</tr>
<tr>
<td>Dikopshof</td>
<td>Germany</td>
<td>31 years</td>
<td>Measured every hour, averaged to daily</td>
<td>R, W, AT, H, SR</td>
</tr>
<tr>
<td>Nossen</td>
<td>Germany</td>
<td>31 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jokioinen</td>
<td>Finland</td>
<td>31 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lleida</td>
<td>Spain</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where:
R – precipitation [mm],
W – wind speed [m/s],
AT – air temperature at the height of 2 m [°C],
H – relative air humidity at the height of 2 m [%],
SR – global shortwave radiation at the height of 2 m [W/m²].


Locations of weather stations used in study on climate dynamics across European transect

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Climate dynamics analysis across European transect

Multifractal spectra of meteorological time series recorded at station located in Jokioinen, Finland for 1980-2001 period (left column) and 2002-2010 period (right column). $f(\alpha)$ is singularity spectrum and $\alpha$ is singularity strength. Panels show original (upper row), shuffled (middle row) and surrogate data (bottom row) for 1980-2001 (left column) and 2002-2010 (right column).

- If monofractal then LRC dominate, no change - LRC has no effect on multifractality
- AAFT (amplitude adjusted Fourier transformation), no change - broadness of PDF has no impact on multifractality

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Multifractal spectra of meteorological time series recorded at station located in Lleida, Spain for 1980-2001 period (left column) and 2002-2010 period (right column). $f(\alpha)$ is singularity spectrum and $\alpha$ is singularity strength. Panels show original (upper row), shuffled (middle row) and surrogate data (bottom row) for 1980-2001 (left column) and 2002-2010 (right column).
Climate dynamics analysis across European transect

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Absolute difference of Hurst exponents for original and shuffled data $|h(q) - h_{shuf}(q)| = |h_{cor}(q)|$ and original and surrogate data $|h(q) - h_{sur}(q)| = |h_{PDF}(q)|$ as a function of $q$ for studied meteorological time series, Jokioinen (two upper plots, for 1980-2001 period and 2002-2010 period) and Lleida (two bottom plots, for 1980-2001 period and 2002-2010 period).
Comparison of computed parameters (dimensionless) of multifractal spectra for the data divided into 2 separate periods: 1980 to 2001 and 2002 to 2010. $\alpha_0$ is an $\alpha$-value corresponding to the maximum of the $f(\alpha)$ function; $a_s$ is asymmetry parameter and $w$ means the width of the multifractal spectrum.

slightly larger, almost no change
still right skewed, but tends to symmetric shape, fine structures still dominate, but more extreme events occurs
less developed multifractality
Effect of spatial averaging on multifractal properties of meteorological time series

Investigated region – the state of North-Rhine Westphalia (NRW).

(A) Location in Europe. Thick white line: NRW; thin white line: Germany. (B) Elevation of NRW. Each colour represents one quartile of the elevations in NRW at 1 km resolution.

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Effect of spatial averaging on multifractal properties of meteorological time series

Multifractal properties asymmetry ($a_s$), width ($w$) and $\alpha_0$ of precipitation, mean temperature and global radiation time series spatially aggregated (1 km grid).

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Effect of spatial averaging on multifractal properties of meteorological time series

Multifractal properties asymmetry ($a_s$), $\alpha_0$ and width ($w$) of precipitation time series spatially aggregated from 1 km to 10, 25, 50 and 100 km resolution (from left to right).

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Effect of temporal averaging on multifractal properties of meteorological time series

where:
R – precipitation [mm],
W – wind speed [m/s],
ST – soil temperature at the depth of 5 cm [°C],
AT – air temperature at the height of 2 m [°C],
ISR – incoming shortwave radiation at the height of 2 m [W/m²].
Effect of temporal averaging on multifractal properties of meteorological time series

Multifractal spectra of meteorological time series based on hourly data (left column) and daily data (right column). Original data spectra are presented in plots A and B; shuffled data plots are in plots C and D; and surrogated data are in plots E and F.
Effect of temporal averaging on multifractal properties of meteorological time series

Absolute difference of Hurst exponents for original and shuffled data $|h(q) - h_{\text{shuf}}(q)| = |h_{\text{cor}}(q)|$ and original and surrogate data $|h(q) - h_{\text{sur}}(q)| = |h_{\text{PDF}}(q)|$ as a function of $q$ for studied meteorological time series, hourly (upper plot) and daily (bottom plot).
Main message

1. Multifractal spectrum of the precipitation time series differs from the spectra of other climate variables. Its multifractality is influenced by a broad probability density function to a greater extent than for other climate variables, for which the long-range correlations prevail.
2. The specific multifractal properties of the precipitation time series may increase its vulnerability to changes in climate dynamics.
3. Spatial data aggregation affects scaling properties of meteorological time series, narrowing not only spatial, but also the temporal variability of the data. Multifractal analysis shows that precipitation time series is most sensitive for this narrowing.
4. Daily aggregation results in losing some unique multifractal features of hourly time series, as the input of both multifractality sources (LRC and PDF) into multifractality is smaller than before the aggregation.
5. Further studies are needed! (longer time series, more stations = more climatic zones analyzed) with the use of not only MF-DFA, but also chaos theory and other methods.

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THANK YOU FOR YOUR ATTENTION!